The Incentive for Separation: 
Job Training and Buyout Options

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Abstract

This study examines how job training combined with a buyout contract may induce workers with a poor fit with a firm to leave as a means of enhancing the firm’s screening efficiency and value. In many cases, neither workers nor the employing firm have information about their fit despite the fact that the fit may be an important determinant of employee performance. Job training offers an opportunity for the worker to gauge their fit. Although the worker is the only party privy to such learning about the fit, the firm can design more efficient contracts and selectively motivate a trained worker with a signal of a high fit.

Moreover, a firm’s buyout offer can improve the screening efficiency by providing an incentive for the worker to leave voluntarily if he observes a low-fit signal. In particular, the firm can offer a buyout to a worker at a given price provided the worker chooses to take that option. However, the buyout contract may be too aggressive and encourage even workers with a high fit to leave in response to an imperfect signal from job training. Considering the overall trade-off, this research shows that a buyout contract can increase a firm’s value as long as the precision of the signal from the job training is not too low.

Keywords: Adverse Selection; Compensation; Job Training.
1 Introduction

Hiring a right worker is vital to a firm’s success, and for the hiring decision, firms set up screening processes to gauge the quality of workers. Typically, research has viewed a worker’s quality or fit to be information known to the worker but not to the employer and substantial effort has been devoted to identifying the mechanisms to extract such private information. In practice, however, it is often the case that the worker as well does not have a great sense of the quality or fit before joining a firm. In such circumstances, this paper demonstrates that job training combined with a labor buyout contract helps a firm hire the right workers. The conventional wisdom is that a worker’s private information causes a disadvantage to a firm by creating information asymmetry. However, this paper shows a firm needs to help a worker learn his private information through job training, thereby allowing a firm to design a more efficient compensation scheme. In addition, it has long been recognized that a guaranteed payment in the form of a fixed payment cannot work as an incentive device, but this study suggests that a fixed buyout price can serve as a contracting tool when the hidden knowledge problem exists at the interim stage.

Widespread research has considered the role of job training in mitigating information asymmetry between a new employee and a firm (e.g., Acemoglu and Pischke (1998), Autor (2001), Bac (2000), and Arya and Mittendorf (2006)). Bac (2000) shows that job training reveals workers’ poor productivity, and Autor (2001) examines why many firms offer free general training to employees despite the sunk up-front training costs. Arya and Mittendorf (2006) show that job training opportunities can screen workers with low ability because they can gauge employee learning. Most of the prior research in the area of job training holds that a worker’s individual ability is the most important determinant of firm productivity and shows that job training can identify strong workers who exhibit their skills. However, in many cases the fit between the worker and the firm may be more important than individual skill; even a worker with excellent skills cannot perform well if those skills are not needed by the firm.
Also, the only way to learn the fit is to experience it in the firm: the fit is realized after a worker has been employed. Thus, the critical variable of the fit between a worker and a firm is not observable at the initial contracting stage. In such circumstances, this paper examines how job training combined with a firm’s labor buyout may provide a screening device that firms can use to determine a worker’s fit with the firm. Interestingly, unlike the previous literature, the job training in this paper creates information asymmetry between a worker and a firm by helping a worker learn about his private information.

Labor buyouts are common in practice but their role as a screening device has not been closely examined in previous research. Demski and Sappington (1991) investigate a variation of labor buyouts and show that a labor buyout can resolve a double moral hazard problem. In their paper, by the buyout contract, if a worker (agent) anticipates that he may have to buy out a firm by a principal’s request, he exerts high effort to raise the firm’s value, thus causing the principal to exert high effort. Unlike the worker’s buyout in Demski and Sappington (1991), this paper focuses on a firm’s labor buyout following common practices: firms often buy out workers’ positions in the firm by offering labor buyout programs at times to reduce labor costs. In particular, the firm offers a buyout to a worker at a given price provided the worker chooses to take that option. In the common practice, this paper posits that a buyout offer after job training can be a natural screening mechanism in the presence of a hidden knowledge problem.

In detail, a worker performs better when the fit with the firm is higher, but at the hiring and contracting stage, neither the worker nor the firm has information about the fit. The fit is realized this only after a worker completes the training or begins working for the firm. Because a principal and an agent (worker) are symmetrically uninformed at the contracting stage, this paper studies the hidden knowledge problem: the agent acquires private information after the contracting stage. This setting follows the previous literature (Holmstrom (1999), Hermelin and Weisbach (1998) and Hermelin (2005)) in that the principal and the agent are symmetrically uninformed and an ability in a particular job (a fit or a quality of match between
an agent and a firm) is considered. In that sense, the fit in this paper can be also interpreted as a firm-specific ability which can be realized after a worker joins a firm.

The late realization of the fit information prevents the firm from hiring the best-fit worker, and many workers choose to stay in the position regardless of the level of fit until a more attractive outside opportunity becomes available. This retention is costly for the firm because the firm must motivate both high- and low-fit workers to exert high effort despite the fact that the efforts of a worker with a low fit will not be productive. In this case, job training provides the worker with a chance to learn his fit with the firm before making an effort choice despite the fact that this information is not perfect and is observable only to the worker. The signal of fit is a worker’s private information, unknown to the firm, but the firm still can use it by designing a selective incentive scheme which induces only a worker with a high-fit signal to exert high effort. Thus job training allows a firm to design more efficient incentive scheme, which improves the firm’s profit.

However, job training cannot achieve a screening purpose at this point because the fit information from job training continues to be private information to the worker. The worker does not leave the firm voluntarily, even if he knows that he may be a low-fit worker, as he can earn information rent by staying. In this case, the labor buyout encourages low-fit signal workers to accept a fixed buyout price and leave, which is less expensive for the firm compared to retaining those workers and paying them compensation. If the worker takes the buyout offer, he leaves the firm with a fixed payment (a predetermined buyout price). If the worker does not take the offer, he stays with the firm and exerts effort. It is always best for a worker with a low-fit signal to leave with the buyout price, which is greater than the payoff from staying. Thus, the firm can improve its profit by using the worker’s private information, saving compensation costs and replacing the low-fit worker with a new worker who is more productive on average. The benefit from the buyout contract is greatest with a high precision of fit signals such that the firm does not suffer the cost of incorrectly pushing out a worker with a high fit.
Labor buyouts are prevalent in practice. However, a firm’s use of labor buyouts as a screening device has received surprisingly little attention from researchers. Nonetheless, the basic premise here has substantial support in practice. At Zappos, an online shoe retailer, new call center employees undergo a few weeks of intensive training and then receive an offer of $1,000, on top of what they have earned if they want to quit (New York Times, May 24, 2008). Employees who do not fit with the firm and who likely are not committed to it take the money and quit. In this case, the offer represents the firm’s labor buyout, and $1,000 is the buyout price. The training period enables employees to learn about their fit with the firm; if an employee discovers that he is not a good fit, he takes the buyout price and leaves. Without such compensation to quit, however, an employee with a low fit would stay to continue receiving compensation and satisfy his or her reservation wage. Employees leave a firm only when the incentive for leaving is sufficient and will not quit without any compensation.

Early retirement is another form of labor buyouts. It is a tool that many firms rely on to eliminate unproductive employees. Firms grant compensation to leave, and employees with relatively less fit or commitment accept the compensation to leave before they are scheduled to retire. As another example, Intel has made use of buyout offers to new recruits. The chipmaker wanted to reduce the number of workers, and new hires could receive a bonus to quit if they decided not to join the firm (Forbes.com, April 26, 2001). The main purpose of this early buyout was to reduce staffing costs by discouraging new hires from getting into the firm. However, it could be a good screening opportunity for the firm to select committed workers. That is, new recruits who were committed to a job in the firm would have not taken the offer and instead would have taken the risk by joining the firm. Therefore, using an early buyout offer, the firm could jettison uncommitted workers while reducing their compensation costs. These very common practices have escaped the scrutiny of serious academic study. Therefore, this research sheds light on the use of the feature of labor buyout agreements as a screening tool.
In terms of related literature, this paper is linked to the agency contracting literature which examines the buyout agreement between a principal and an agent. Demski and Sappington (1991) utilize labor buyouts as an incentive scheme as in this paper. They show that a buyout contract can motivate an agent to exert high effort because the agent may have to buy out a firm at a predetermined buyout price. The current paper also examines the benefit of such a buyout contract and shows the potential value of prespecified fixed buyout prices in resolving the hidden knowledge problem. Unlike the previous paper, this research considers a firm's buyout rather than a worker's buyout and focuses on a firm's screening process in the presence of a hidden knowledge problem.

This paper builds on previous research regarding the role of job training. Acemoglu and Pischke (1998) examine why firms offer general skill training, finding that firms obtain ex post informational monopsony power through the information they acquire during the training period. Bac (2000) investigates the conflict between efficient screening and employee acquisition of firm-specific skills, positing that screening efficiency may have to be sacrificed to some extent in the employer's postcontractual incentives. By investigating firms that offer free general training to workers, Autor (2001) shows both theoretically and empirically that a firm can attract high-quality employees with free training; such training allows employees to self-select, even as the firm observes the employees' skill. Arya and Mittendorf (2006) show that a job rotation program can match compensation to an employee's truthful ability. These prior studies focus on the worker's ability, rather than fit, assuming that ability can be observed through job training. In contrast, this study addresses fit, which is often unobservable to the firm during job training.

This paper pertains to a stream of accounting literature which examines the effect of providing an agent with predecision information. As one of the early works in the literature, Christensen (1981, 1982) analyzes situations in which an agent can hold private information before making an effort choice, showing that a principal can be worse off due to an agent's
predecision information. The current paper also examines the effect of an agent’s private information before making an effort choice. However, the primary emphasis in the previous papers was on the moral hazard problem, whereas, the current paper focuses on the adverse selection problem, especially when the agent’s private information is acquired after the contracting stage.

Additionally, the current research is closely related to Penno (1984) in that a firm (the principal) intentionally creates an information asymmetry by providing an opportunity for a worker (the agent) to learn private information. Moreover, communication between a firm and a worker is not considered. Penno (1984) shows that allowing an agent to access to managerial accounting information before determining his level of effort makes the principal strictly better off. On the other hand, the current research demonstrates that job training enables a worker to learn his private information and that a firm can use the private information in designing more efficient compensation and in screening workers.

Furthermore, this research is in line with the literature that examines the hidden knowledge problem. Levitt and Snyder (1997) study incentive compensation, which, can motivate a manager who has already exerted effort to report a private signal about eventual project outcomes. Although this study also considers incentive compensation in the presence of hidden knowledge, the focus is on an initial screening mechanism such that the worker is induced to reveal his private information before he exerts effort.

Vaysman (2006) shows that paying managers shut-down bonuses can encourage privately informed managers to make the optimal abandonment decision for a project. On the other hand, this paper examines how job training and buyout options encourage a worker to find his private information (a fit) and to leave voluntarily if he has a low fit with a firm.

Inderst and Muller (2010) show that it can be best to reward CEOs through a steep contingent payment rather than simply using a severance payment when a firm replaces a badly matched CEO. Their paper is different from the current work in two key aspects. First, whereas
their investigation pertains to CEO replacement, this study focuses on a lower hierarchical level, specifically workers, and therefore includes job training as an important mechanism. Second, in their paper the CEO already has affected the firm’s value before replacement, as their model assumes that initially the best CEO candidate is hired exogenously and that the CEO exerts effort before knowing his or her type. In contrast, in the current study, the screening occurs at the hiring stage and a worker knows the fit level before exerting effort. Thus, the firm value is determined by the efforts of only selected workers based on their fit.

Finally, Van Wesep (2010) studies the use of signing bonuses to attract employees whose fit with the firm is uncertain. A signing bonus provides a credible signal to the worker of a possible high match. In such a case, the singing bonus persuades the employee to join the firm. On the other hand, the current study examines the role of the buyout contract in screening employees’ fit.

The remainder of this article consists of seven sections. Section 2 describes the model and provides a benchmark case, while section 3 describes cases without and with job training. Section 4 investigates the benefit of a buyout contract as an incentive device, and this is followed by a numerical example in section 5 to explain the main results. Section 6 considers skill learning and endogenous firm value as extensions, while section 7 concludes the paper.

2 Setup

2.1 Model

There exists a firm which considers a pool of workers. The firm hires each worker to generate additional firm value. At the contracting stage, information about the fit between the worker and the firm, which determines the firm’s value, is unavailable. This fit, realized only after the worker is employed in the firm for a given period of time, consists of two types, $\theta \in (\theta_L, \theta_H)$, such that $\theta_H$ is more productive than $\theta_L$. The probability of a high fit is equal to $\mu \in (0, 1)$. The fit is observable to only the worker and is the worker’s private information.
The firm can offer job training, which the worker uses to receive a signal \((\sigma)\) of fit. The signal of fit has two realizations, \(\sigma \in (\sigma_L, \sigma_H)\). The signal is informative but imperfect, such that

\[
\Pr(\sigma_H | \theta_H) = \lambda \quad \text{and} \quad \Pr(\sigma_L | \theta_H) = 1 - \lambda; \\
\Pr(\sigma_L | \theta_L) = \lambda \quad \text{and} \quad \Pr(\sigma_H | \theta_L) = 1 - \lambda,
\]

where \(\lambda \in (\frac{1}{2}, 1]\).

A firm’s terminal value, \(v \in \{v_L, v_H\}\), depends on both the fit \((\theta_i)\) and the worker’s effort level \((e \in \{e_L = 0, e_H = 1\})\). The costs of effort are \(C(e = 0) = 0\) and \(C(e = 1) = c > 0\).

Then, the probability structure for a worker to generate each firm value is as follows:

\[
\Pr(v_H | \theta, e) = \begin{cases} 
pe + q(1 - e) & \text{if } \theta = \theta_H \\
0 & \text{if } \theta = \theta_L
\end{cases}; \\
\Pr(v_L | \theta, e) = 1 - \Pr(v_H | \theta, e),
\]

where \(p, q \in \left(\frac{1}{2}, 1\right)\) and \(p > q\).

The probability of generating each firm value is a function of a worker’s fit \((\theta)\) and effort \((e)\). That is, if a worker with a high fit \((\theta_H)\) exerts a high level of effort \((e_H = 1)\), the probability of a high firm value \((v_H)\) is \(p\). If the worker with a high fit exerts a low level of effort \((e_L = 0)\), the chance of a high firm value becomes lower from \(p\) to \(q\). On the other hand, a low fit \((\theta_L)\) leads to a low firm value \((v_L)\), regardless of the effort level. This is the main reason the firm prefers a worker with a high fit \((\theta_H)\). This model will be extended in the later section (Section 6.1) in which even a worker with a low fit can generate a high firm value if he exerts a high level of effort.

Both a firm and a worker are risk-neutral. A worker’s utility is \(u(\theta) = t - c\), and a firm’s profit is \(\Pi = v - t\), where \(t \in \{t_L, t_H\}\) and \(t_H (t_L)\) is compensation for a high (low) firm value.

The buyout options are offered to any worker who joins a firm but they expire after job training. The worker may take the buyout option after observing a fit signal from job training and before exerting effort. If a worker takes the buyout option, he leaves the firm with the
predetermined buyout price $K$, as well as a reservation wage $U$ in the outside job market, which is normalized to zero for simplicity. Furthermore, with the buyout contract after the fit is realized, the worker’s interim expected utility is

$$E[u|\sigma_H] = \max\{K, \Pr(\theta_H|\sigma_H)(pt_H + (1-p)t_L) + (1-\Pr(\theta_H|\sigma_H))t_L - c\};$$

$$E[u|\sigma_L] = \max\{K, \Pr(\theta_L|\sigma_L)t_L + (1-\Pr(\theta_L|\sigma_L))(qt_H + (1-q)t_L)\},$$

such that the worker prefers to exert high (low) effort if a high (low) fit signal is realized, which is true in equilibrium. The worker decides whether to take the buyout offer by comparing the payoffs. If the worker does not take the option, the option immediately expires and the worker receives $t_i$ by exerting effort at the end of the period. If a worker leaves, the firm finds a new employee to realize its value, $V$, which is exogenously determined from a range between $v_L$ and $v_H$: $V \in (v_L, v_H)$. This assumption for the exogenous value of $V$ will be relaxed in the later section (Section 6.2) by endogenizing the value of $V$.

Figure 1 shows the time line.

The time line is therefore as follows:

1. A worker enters into a contract with a firm.
2. The firm offers job training.
3. A signal ($\sigma$) of fit ($\theta$) is realized from the job training.
4. The worker decides whether or not to take the buyout option.
5. If the worker takes the buyout option, he leaves the firm with $K$. Then, the firm finds a new worker to realize its value, $V$.
6. If the worker does not take the buyout option, he stays and decides whether he exerts effort or not.
7. The firm’s profit and the worker’s compensation are realized.

Fig. 1. The Sequence of Events
(6) If the worker leaves, the firm finds a new worker to realize its value, \( V \).

(7) If the worker does not take the buyout option, he stays with the firm and decides whether or not to exert effort.

(8) The firm’s value (\( v \)) is realized, and the worker receives compensation, \( t \).

Thus the worker exerts effort and influences the firm’s value after job training, as he undertakes his job duties after receiving training. To avoid a trivial result, it is assumed that the firm wants a high-fit worker to exert high effort. The sufficient condition for this assumption is that the difference between \( v_H \) and \( v_L \) is sufficiently large.

### 2.2 Benchmark

Consider a setting in which the fit between a worker and a firm is publicly observed after job training. That is, at the contracting stage, both a worker and a firm do not know the fit. However, the worker receives job training after accepting a contract, and after job training, a perfect signal (\( \lambda = 1 \)) of the fit can be observed by both the worker and the firm. After job training, if the firm observes a low-fit signal, the worker is asked to leave, and only workers with a high fit remain. In this case, a firm designs a contract which induces a high level of effort from a worker. The firm’s contracting problem is presented as follows:

\[
\begin{align*}
\text{Max} & \quad \mu (p(v_H - t_H) + (1 - p)(v_L - t_L)) + (1 - \mu)V \\
\text{s.t.} & \\
\mu(pt_H + (1 - p)t_L - c) \geq 0 \quad \text{(EIR)} \\
p t_H + (1 - p)t_L - c \geq 0 \quad \text{(IIR)} \\
p t_H + (1 - p)t_L - c \geq q t_H + (1 - q)t_L \quad \text{(IC)} \\
q t_H + (1 - p)t_L \geq 0 \quad \text{(LL)} \\
t_H, t_L \geq 0
\end{align*}
\]

In the objective function, the probability that a worker has a high fit is \( \mu \). The firm can generate the expected value, \( p v_H + (1 - p)v_L \) by motivating a high-fit worker to exert high
effort. However, a firm does not hire a worker with a low fit with a probability \((1 - \mu)\), and the firm’s value of replacing this worker with another worker is \(V\). When the fit between the firm and the worker is public information, the firm’s sole objective is to motivate the worker to exert high effort to increase the chances of generating a high firm value, \(v_H\).

The ex ante individual rationality constraint (EIR) ensures that the contract provides a worker with at least a reservation wage \((\bar{U})\), which is normalized to zero. Ex ante, neither the firm nor the worker knows whether the worker has a high or low fit. Because a low fit, as observed through job training, makes the worker leave without exerting effort and receiving compensation, the (EIR) includes a worker’s expected compensation only for a high fit. Unless the firm provides enough expected compensation to offset the worker’s concern about a realization of a low fit, the worker will not enter into a contract, as he or she still may be asked to leave after job training. The (EIR) constraint makes a worker participate in job training to secure expected compensation better than a reservation wage ex ante. However, as explained below, this (EIR) constraint is not binding in equilibrium.

Moreover, the incentive compatibility constraint (IC) motivates a high-fit hired worker to exert high effort. The (LL) constraint reflects the worker’s limited liability.

In this program, the (IC) constraint is binding to induce a worker’s high effort and the optimal values for \(t_H\) and \(t_L\) can be derived from the (IC) constraint. Moreover, the optimal values satisfy the (EIR) constraint. This process yields a benchmark result, as summarized in Lemma 1. (All proofs appear in the Appendix.)

**Lemma 1** When the fit between a worker and a firm is public information after job training, the equilibrium outcomes are as follows:

\[
t_L^* = 0; \quad t_H^* = \frac{c}{p - q}; \quad \Pi_F = \mu \left( pv_H + (1 - p)v_L - \frac{c}{p - q} \right) + (1 - \mu)V
\]

and the first-best solution is obtained.
3 Job Training

This section examines the benefits and costs of job training. Consider a setting in which a worker’s fit is not publicly observable even after job training, but instead is the worker’s private information. The worker receives an imperfect signal of his own fit through job training, and the firm designs a contract to screen out low-fit-signal workers and to motivate high-fit-signal workers to stay and exert high effort.

3.1 No Job Training

Before investigating this role of job training, consider a case without job training, which helps make the incremental benefit of job training more explicit in the following sections.

Without job training, a worker cannot observe the fit before exerting effort for the firm, nor can the worker leave voluntarily. Therefore, both high- and low-fit workers join the firm, and the firm must motivate all workers to exert high effort, even if it realizes that efforts by a low-fit worker will be fruitless.

Technically, in the absence of job training, a firm cannot design a contract with an interim signal, which leaves no interim constraints because the signal of fit does not arrive without job training. In this situation, the firm’s contracting problem is as follows:

\[
\begin{align*}
\max_{t_H, t_L} & \quad \mu (p(v_H - t_H) + (1 - p)(v_L - t_L)) + (1 - \mu)(v_L - t_L) \\
\text{s.t.} & \quad \mu E[u(\theta_H, e_H)] + (1 - \mu)E[u(\theta_L, e_H)] \geq 0 \quad \text{(EIR)} \\
& \quad \mu E[u(\theta_H, e_H)] + (1 - \mu)E[u(\theta_L, e_H)] \geq \mu E[u(\theta_H, e_L)] + (1 - \mu)E[u(\theta_L, e_L)] \quad \text{(EIC-ALL)} \\
& \quad t_H, t_L \geq 0, \quad \text{(LL)}
\end{align*}
\]

where \( E[u(\theta_H, e_H)] = pt_H + (1 - p)t_L - c \) and \( E[u(\theta_L, e_H)] = t_L - c \).
In the objective function, the firm’s probability of having a low terminal value \( v_L \) increases from \( \mu(1-p) \) in the previous first-best case to \( \mu(1-p) + (1-\mu) \) here because it cannot discourage a low-fit worker from exerting high effort, which is costly to the firm. That is, the effort of the low-fit worker provides a firm a value of only \( v_L \); note also that \( \Pr(v_L|\theta_L, \cdot) = 1 \).

Because neither a firm nor a worker knows the fit until the worker exerts effort, the firm has to motivate both types of workers through the contract. This constraint (EIC-ALL) motivates both of types of workers to choose a high effort level. Based on the program, we obtain the following result in Proposition 1.

**Proposition 1**  
(1) Without job training, the equilibrium outcomes are as follows:

\[
t^*_L = 0; \quad t^*_H = \frac{c}{\mu(p-q)}; \quad \Pi_N = \mu(pv_H + (1-p)v_L) + (1-\mu)v_L - \frac{cp}{p-q}.
\]

(2) Without job training, the firm’s profit is always lower than the benchmark profit, i.e.,

\[
\Pi_N - \Pi_F = -\frac{(1-\mu)((p-q)(V - v_L) + cp)}{p-q} < 0.
\]

The absence of job training does not allow either the worker or the firm to know the fit before the worker exerts efforts, which creates two types of costs. First, the firm has a higher likelihood of generating a low firm value \( v_L \), as a low-fit worker does not leave but instead remains and exerts efforts leading only to the firm value of \( v_L \). Second, without job training, the firm must commit more expenditures to motivating all workers instead of designing a selective contract that motivates only high-fit workers. The firm thus compensates even the fruitless efforts of low-fit workers ex ante, which increases its compensation costs. These two costs lower the firm’s profit, as stated in Proposition 1. The next section shows that job training can improve the firm’s profit by reducing compensation costs.

### 3.2 Job Training

Consider a job training opportunity for a worker. The worker joins the firm and receives job training for a given period. The job training generates the signal \( \sigma \) of the fit \( \theta \) with
precision \( \lambda \in (\frac{1}{2}, 1] \). However, the fit signal can be observed only by the worker and is his private information: the fit is often not observable to the firm when it is a large organization. This section shows that job training can improve a firm’s profit while not offering a screening purpose at this point because both a high-fit worker and a low-fit worker remain on the job. The low-fit worker does not want to leave a firm without an incentive.

In detail, in the presence of job training, the compensation schemes, \( t_H \) and \( t_L \) are chosen as follows:

\[
\begin{align*}
\max_{t_H, t_L} \mu (\lambda (p(v_H - t_H) + (1-p)(v_L - t_L)) &+ (1 - \lambda)(q(v_H - t_H) + (1-q)(v_L - t_L))) + (1 - \mu)(v_L - t_L) \\
\text{s.t.} \quad\mu (\lambda (pt_H + (1-p)t_L - c) &+ (1 - \lambda)(qt_H + (1-q)t_L)) + (1 - \mu)t_L \geq 0 \quad \text{(EIR)} \\
E[u|\sigma_H, e_H] &\geq 0 \quad \text{(IIR-H)} \\
E[u|\sigma_H, e_H] &\geq E[u|\sigma_H, e_L] \quad \text{(IC-H)} \\
t_H, t_L &\geq 0, \quad \text{(LL)}
\end{align*}
\]

where \( E[u|\sigma_H, e_H] = \Pr(\theta_H|\sigma_H) (pt_H + (1-p)t_L) + (1 - \Pr(\theta_H|\sigma_H))t_L - c \) and \( E[u|\sigma_H, e_L] = \Pr(\theta_H|\sigma_H) (qt_H + (1-q)t_L) + (1 - \Pr(\theta_H|\sigma_H))t_L \). \( \Pr(\theta_H|\sigma_H) \) is the worker’s posterior belief that he has a high fit when \( \sigma_H \) is realized and the posterior belief structure for each signal is as follows:

\[
\begin{align*}
\Pr(\theta_H|\sigma_H) &= \frac{\mu \lambda}{\mu \lambda + (1-\mu)(1-\lambda)}; \quad \Pr(\theta_L|\sigma_H) = \frac{(1-\mu)(1-\lambda)}{\mu \lambda + (1-\mu)(1-\lambda)}, \\
\Pr(\theta_L|\sigma_L) &= \frac{(1-\mu)\lambda}{(1-\mu)\lambda + \mu(1-\lambda)}; \quad \Pr(\theta_H|\sigma_L) = \frac{\mu(1-\lambda)}{(1-\mu)\lambda + \mu(1-\lambda)}.
\end{align*}
\]

Similar to the case without no job training case, the (EIR) constraint considers both high and low fits because, ex ante, a worker cannot know his fit. A worker who receives a signal of a low fit (\( \sigma_L \)) prefers to exert low effort as it is very likely that this effort will be useless, which is true in equilibrium (see the Appendix).
The solutions have the (IC-H) constraint binding and the optimal solutions are as follows:

\[ t_L^* = 0; \quad t_H^* = \frac{c}{\Pr(\theta_H|\sigma_H)(p - q)} = \frac{c(\mu \lambda + (1 - \mu)(1 - \lambda))}{\mu \lambda (p - q)}. \]

Job training creates an opportunity for the worker to learn his type and provides the firm with the means to selectively motivate workers. This selective motivation also enables the firm to reduce its compensation costs because it does not compensate the fruitless efforts of low-fit workers.

Comparing the expected compensation under the job training \((C_J)\) with that in the absence of job training \((C_N)\) indicates:

\[ C_N - C_J = \Delta C = \frac{cp}{p - q} - \frac{c(\lambda(p - q) + q)(1 - \mu + \lambda(2\mu - 1))}{\lambda(p - q)} > 0, \]

which implies that the compensation cost is lower with job training. \(\Delta C\) is always positive because \(\Delta C\) is positive when \(\mu = 1\); it increases as \(\mu\) decreases. In detail, \(\mu\) represents the likelihood that a worker has a high fit; as \(\mu\) decreases, the chance that a worker has a low fit increases and screening becomes more important. \(\lambda\) reflects the precision of the signal provided by job training. The comparison reveals that as the importance of screening increases and the signal becomes more precise, the benefits of job training, in terms of lowering compensation costs, increases.

However, even in the presence of job training, a worker with a low-fit signal remains with the firm to receive information rents for the following reason. The signal from job training is imperfect according to the precision \(\lambda \in \left(\frac{1}{2}, 1\right]\); thus, there is a chance that a worker who receives a low fit signal actually has a high fit. Thus, a worker who receives a low-fit signal prefers to stay with the firm but does not exert high effort, which provides him with the expected compensation, as follows:

\[ E[u|\sigma_L] = \Pr(\theta_L|\sigma_L)t_L + (1 - \Pr(\theta_L|\sigma_L))(qt_H + (1 - q)t_L) \]

\[ = \frac{cq(1 - \Pr(\theta_L|\sigma_L))}{\Pr(\theta_H|\sigma_H)(p - q)} = \frac{cq(1 - \lambda)(\mu \lambda + (1 - \mu)(1 - \lambda))}{(p - q)\lambda(1 - \mu \lambda - (1 - \mu)(1 - \lambda))} > 0. \]
If the signal is perfect ($\lambda = 1$), any worker receiving a low signal leaves. The imperfect signal instead causes workers to wait for a final outcome to confirm the fit and a firm cannot screen out workers by job training only.

Even if a worker with a low-fit signal actually has a high fit, retaining this worker has a limited effect on a firm’s value because any worker with a low-fit signal prefers not to exert high effort. The firm’s chance of generating a high terminal value ($v_H$) is thus lower ($q < p$), even if the worker has a high fit. This efficiency loss due to an imperfect signal represents a downside of job training. Proposition 2 therefore reflects the trade-off associated with job training to determine the equilibrium outcomes and results.

**Proposition 2** (1) With job training, the equilibrium outcomes are as follows:

$$t_L^* = 0; \quad t_H^* = \frac{c(\mu\lambda + (1 - \mu)(1 - \lambda))}{\mu\lambda(p - q)};$$

$$\Pi_J = \mu(\lambda p + (1 - \lambda)q)(v_H - v_L) + v_L - \frac{c(\lambda p + (1 - \lambda)q)(\mu\lambda + (1 - \mu)(1 - \lambda))}{\lambda(p - q)}.$$ 

(2) Job training improves a firm’s profit when screening becomes more important, i.e.,

$$\Pi_J - \Pi_N \geq 0 \quad \text{if} \quad \mu \leq \mu^J,$$

where $$\mu^J = \frac{c(p\lambda^2 - q(1 - \lambda)^2)}{(p - q)^2(v_H - v_L)(1 - \lambda)\lambda + c(2\lambda - 1)(p\lambda + q(1 - \lambda)).}$$

(3) No worker voluntarily leaves a firm regardless of the signal from job training.

According to Proposition 2, the firm’s profit is greater with job training when the likelihood that a worker has a high fit is not exceedingly high. That is, because the signal from job training is not perfect, a worker with a low-fit signal may have a high fit. This type I error represents the cost of job training because this worker exerts low effort. The trade-off between reducing compensation costs and the cost due to the type I error determines whether the firm offers job training or not.

The precision of the signal from job training also affects the size of the information rent for workers who receive a low-fit signal. When the signal becomes more precise, the worker
gains confidence in his fit level after job training; ex ante, the worker’s compensation risk then decreases. The worker worries less about receiving low compensation even after exerting a high effort, which may occur if the worker obtains a high-fit signal but actually has a low fit. As the signal becomes more precise, the risk becomes even smaller, and the firm can pay less \( t^*_H \), which also decreases the information rent for a low-fit worker. Note that \( \frac{\partial E[u\mid \sigma_L]}{\partial \lambda} = -\frac{cq(1-\mu)(\mu+2\lambda(1-\lambda)(1-2\mu))}{(p-q)\lambda^2(\mu+\lambda-2\mu\lambda)^2} < 0 \). Therefore, as job training becomes more precise, the worker’s information rent decreases. This result is confirmed in Corollary 1.

**Corollary 1** As the job training is more precisely designed, the worker’s information rent decreases, i.e., \( \frac{\partial E[u\mid \sigma_L]}{\partial \lambda} < 0 \).

Thus, job training improves the firm’s profit under some conditions but still does not achieve the screening purpose because the signal from job training remains the worker’s private information. The next section details how a buyout option coupled with job training can extract the worker’s private information and screen out low-fit workers.

## 4 Buyout Options

This section considers a buyout option offered to a worker and examines how a labor buyout affects the screening efficiency of job training. As the previous section shows, a worker does not voluntarily leave a firm, even after receiving a signal of low fit, because he still can earn information rents by staying. However, the buyout option may encourage him to leave voluntarily upon the receipt of a low signal; hence, the buyout option eventually enhances the sorting efficiency of job training.

Buyout options are offered to any worker who joins a firm, but they expire after job training. The worker may take the buyout options after observing a fit signal from job training and before exerting effort. If a worker takes the buyout option, he receives a predetermined buyout price \( K \) and leaves. If the worker decides to stay, the buyout option expires, and the worker exerts
effort and receives compensation.

In this sense, the buyout options as a form of interim compensation serves as an additional incentive for a worker to leave voluntarily because the buyout price is greater than the expected compensation that a worker with a low-fit signal would receive by staying. The buyout option thus demands careful design, because an extremely high buyout price may induce even a worker with a high-fit signal to leave. The following contracting problem reflects the combined effect of the buyout option and job training:

$$\max_{t_H, t_L, K} \mu \left( \lambda (p(v_H - t_H) + (1 - p)(v_L - t_L)) + (1 - \lambda)(V - K) \right) + (1 - \mu)(\lambda(V - K) + (1 - \lambda)(v_L - t_L))$$

s.t.

$$\mu E_{\sigma} [u(\theta_H)] + (1 - \mu)E_{\sigma} [u(\theta_L)] \geq 0$$  \hspace{1cm} \text{(EIR)}

$$E [u|\sigma_H, e_H] \geq 0$$  \hspace{1cm} \text{(IIR-H)}

$$E [u|\sigma_H, e_H] \geq E [u|\sigma_H, e_L]$$  \hspace{1cm} \text{(IC-H)}

$$E [u|\sigma_H, e_H] \geq K$$  \hspace{1cm} \text{(NoEXIT-H)}

$$K \geq E [u|\sigma_L, e_L]$$  \hspace{1cm} \text{(EXIT-L)}

$$t_H, t_L \geq 0, \hspace{1cm} \text{(LL)}$$

where

$$E_{\sigma} [u(\theta_H)] = \lambda (p t_H + (1 - p) t_L - c) + (1 - \lambda) K;$$

$$E_{\sigma} [u(\theta_L)] = \lambda K + (1 - \lambda)(t_L - c);$$

$$E [u|\sigma_H, e_H] = \Pr(\theta_H|\sigma_H) (p t_H + (1 - p) t_L) + (1 - \Pr(\theta_H|\sigma_H)) t_L - c;$$

$$E [u|\sigma_H, e_L] = \Pr(\theta_H|\sigma_H) (q t_H + (1 - q) t_L) + (1 - \Pr(\theta_H|\sigma_H)) t_L;$$

$$E [u|\sigma_L, e_L] = \Pr(\theta_L|\sigma_L) t_L + (1 - \Pr(\theta_L|\sigma_L)) (q t_H + (1 - q) t_L).$$

This contracting problem contains two new constraints: (EXIT-L) and (NoEXIT-H). The (EXIT-L) constraint encourages a worker who receives a low-fit signal to leave by promising
the buyout price $K$. To induce the worker to leave, the buyout price should be greater than the expected compensation that a worker with a low-fit signal could receive by staying. This amount determines the lower bound of the buyout price.

In contrast, the (NoEXIT-H) constraint affects a worker with a high-fit signal by encouraging him or her to remain with the firm. Therefore, the buyout price must be smaller than the expected compensation that a high-fit signal worker could earn by exerting high effort, which determines the upper bound of the exercise price. Thus, the two constraints determine the range of the buyout price the firm should use to screen a worker through job training. The binding (IC) constraint and (LL) constraint again thus yield following optimal solutions:

$\ell^*_L = 0; \ell^*_H = \frac{c}{\Pr(\theta_H | \sigma_H)(p - q)} = \frac{c(\mu \lambda + (1 - \mu)(1 - \lambda))}{\mu \lambda (p - q)}$.

With regard to the firm’s objective function, as $K$ becomes lower, the firm’s profit increases. The optimal value for the buyout price, $K$ then depends on the lower bound of the range, $\Pr(\theta_L | \sigma_L)\ell_L + (1 - \Pr(\theta_L | \sigma_L))(q\ell_H + (1 - q)\ell_L)$, which maximizes the firm’s profit while satisfying both the (EXIT-L) and (NoEXIT-H) constraints. Using $\ell^*_L$ and $\ell^*_H$ yields the optimal buyout price:

$K^* = \frac{cq(1 - \Pr(\theta_L | \sigma_L))}{\Pr(\theta_H | \sigma_H)(p - q)} = \frac{cq(1 - \lambda)(\mu \lambda + (1 - \mu)(1 - \lambda))}{(p - q)\lambda(1 - \mu \lambda - (1 - \mu)(1 - \lambda))}$.

In addition, the optimal solutions and buyout price satisfy all other constraints. The main results from this section can therefore be summarized as shown in Proposition 3.

**Proposition 3** (1) When the firm offers buyout options with job training, the equilibrium outcomes are:

$\ell^*_L = 0; \ell^*_H = \frac{c(\mu \lambda + (1 - \mu)(1 - \lambda))}{\mu \lambda (p - q)}$;

$\Pi_B = \mu p \lambda (v_H - v_L) + (\mu \lambda + (1 - \mu)(1 - \lambda))v_L + V(1 - \mu \lambda - (1 - \mu)(1 - \lambda))$

$- \frac{c(\lambda p + (1 - \lambda)q)(\mu \lambda + (1 - \mu)(1 - \lambda))}{\lambda (p - q)}$.

(2) With buyout options, the optimal buyout price is $K^* = \frac{cq(1 - \Pr(\theta_L | \sigma_L))}{\Pr(\theta_H | \sigma_H)(p - q)} = \frac{cq(1 - \lambda)(\mu \lambda + (1 - \mu)(1 - \lambda))}{(p - q)\lambda(1 - \mu \lambda - (1 - \mu)(1 - \lambda))}$. 19
(3) With buyout options, a worker with a low-fit signal ($\sigma = \sigma_L$) voluntarily leaves the firm by taking the buyout option; a worker with a high-fit signal ($\sigma = \sigma_H$) stays with the firm and exerts high effort.

As Proposition 3 indicates, buyout options serve as an incentive device for a low-fit-signal worker to leave. Without buyout options, the worker will stay, regardless of the signal from the job training and wait for a final signal, which comes from the firm’s terminal value when the worker does not exert high effort and thus implies an efficiency loss. The buyout options instead induce a worker who receives a low-fit signal to leave, implying that the firm enjoys a lower probability of a low terminal value, as (1) the remaining workers who received a high-fit signal are more likely to have a high fit and exert high effort, and (2) a low-fit-signal worker is replaced with a new worker who is more productive on average because he may have a high fit with probability $\mu$.

The conventional belief is that a guaranteed payment in the form of a fixed payment cannot be used as an incentive scheme because a certain level of payment is guaranteed for a worker. Despite this conventional belief, as Proposition 3 shows, when a worker’s private information arrives during the interim stage, a guarantee of payment based on the buyout price induces low-fit workers to leave. Thus, the buyout option can screen workers as an incentive device at the interim stage.

In addition, as shown in Proposition 3, the optimal buyout price ($K^*$) varies according to the change of the effort cost ($c$), the productivity ($p$ and $q$), and the signal precision level ($\lambda$). The comparative static is summarized in Corollary 2.

**Corollary 2** (1) The optimal buyout price decreases as the signal from job training becomes more precise, i.e., $\frac{\partial K^*}{\partial \lambda} < 0$.

(2) The optimal buyout price decreases as the productivity of a high effort ($p$) increases, i.e., $\frac{\partial K^*}{\partial p} < 0$. 
(3) The optimal buyout price increases as the productivity of a low effort (q) increases, i.e.,
\[ \frac{\partial K^*}{\partial q} > 0. \]

(4) The optimal buyout price increases as the cost of effort (c) increases, i.e., \[ \frac{\partial K^*}{\partial c} > 0. \]

Thus, according to Corollary 2 (1), the optimal buyout price decreases as the signal from job training becomes more precise (\( \lambda \) increases). If job training is more precise, the worker who receives a low-fit signal anticipates lower information rent. Thus, the firm can induce him to leave with a lower buyout price. In Corollary 2 (2), the optimal buyout price decreases as the productivity of a high effort (p) increases because higher productivity from a high effort decreases the expected compensation for a worker with a low-fit signal in that the firm may pay a lower incentive to induce a high effort and still obtain a high firm value. Corollary 2 (3) states that the optimal buyout price increases as the productivity of a low effort (q) increases, as does the expected compensation of a worker with a low-fit signal when such a worker stays. Therefore, the firm should increase the buyout price to make leaving more attractive to these workers. Finally, Corollary 2 (4) shows that the buyout price increases as the cost of effort (c) increases. A higher cost of effort increases the worker’s compensation with a high firm value, \( t_H^* = \frac{c(\mu \lambda + (1-\mu)(1-\lambda))}{\mu \lambda (p-q)} \). Therefore, staying with the firm becomes more attractive to a worker with a low-fit signal, and the firm should increase the buyout price to induce such workers to leave.

In line with Proposition 3, we confirm that the use of buyout options induces a worker who receives a low-fit signal to leave a firm; only workers with high-fit signals remain and exert high effort. However, these workers are sorted according to an imperfect job training signal rather than by their true fit with the firm, which implies that job training combined with buyout options may exclude a worker who has a high fit but receives an incorrect signal. In this case, the firm suffers an efficiency loss, because it loses a high-fit worker. The buyout option provides a trade-off between the endogenous sorting and type I and II errors, which result from the imperfect signal. With regard to this trade-off, Proposition 4 identifies the
conditions in which buyout options improve the performance of job training.

**Proposition 4** (1) The use of buyout options with the job training improves a firm’s profit compared with the case with no job training when screening becomes more important, i.e.,

\[
\Pi_B \geq \Pi_N \text{ if } \mu \leq \mu^B, \\
\text{where } \mu^B = \frac{\lambda^2(p - q)(V - v_L + c) + cq(2\lambda - 1)}{c(2\lambda - 1)(\lambda p + (1 - \lambda)q) + \lambda(p - q)((1 - \lambda)p(v_H - v_L) + (2\lambda - 1)(V - v_L))}.
\]

(2) The use of buyout options with job training improves a firm’s profit compared with the case with only job training when the precision of the signal from the job training is relatively high, i.e.,

\[
\Pi_B \geq \Pi_J \text{ if } \lambda \geq \lambda^* = \frac{\mu(qv_H + (1 - q)v_L - V)}{\mu(qv_H + (1 - q)v_L - V) + (V - v_L)(1 - \mu)}.
\]

(3) \(\mu^B \geq \mu^J\) if \(\lambda \geq \lambda^*\).

A comparison of the case with no job training to that with job training with buyout options shows that the firm’s profit improves with the use of buyout options when the likelihood of a high-fit worker (\(\mu\)) is lower. The type I and II errors are the costs of buyout options, but the buyout options also enable the firm to increase the chances of sorting out a worker who has a low fit, in which case it generates firm value \(V\) by hiring a new worker. As \(\mu\) decreases, screening becomes more important, and the benefit of buyout options dominates the costs with regard to excluding workers incorrectly. Therefore, the buyout options improve the firm’s profit.

The incremental benefit of buyout options from the case with job training only also shows that the firm’s profit is greater with buyout options as long as \(\lambda \geq \lambda^* = \frac{\mu(qv_H + (1 - q)v_L - V)}{\mu(qv_H + (1 - q)v_L - V) + (V - v_L)(1 - \mu)}\). In this case, \(\lambda\) refers to the precision of the signal from job training. If the signal is more precise than a certain threshold, buyout options improve the firm’s profit by excluding low-fit workers. If the job training is less precisely designed, however, buyout options can be too aggressive in excluding workers and thereby increase the likelihood of error. Therefore, only when the
precision of job training is not too low does the firm’s profit increase with the use of buyout options.

As Proposition 4(2) indicates, the cutoff value, \( \lambda^* \) is endogenously determined by the productivity of a low effort \( (q) \), the firm’s value for replacing a worker \( (V) \), a high firm value \( (v_H) \), and a low firm value \( (v_L) \). When \( \lambda^* \) decreases, the attractiveness of the buyout option increases; the comparative statics with regard to \( \lambda^* \) appear in Corollary 3.

**Corollary 3**

1. The incremental benefit of buyout options decreases as the productivity of a low effort \( (q) \) increases, i.e., \( \frac{\partial \lambda^*}{\partial q} > 0 \).
2. The incremental benefit of buyout options increases as the firm’s value under replacement \( (V) \) increases, i.e., \( \frac{\partial \lambda^*}{\partial V} < 0 \).
3. The incremental benefit of buyout options decreases as \( v_H \) increases, i.e., \( \frac{\partial \lambda^*}{\partial v_H} > 0 \).
4. The incremental benefit of buyout options decreases as \( v_L \) increases, i.e., \( \frac{\partial \lambda^*}{\partial v_L} > 0 \).

As Corollary 3(1) shows, buyout options become less attractive when productivity associated with a low effort \( (q) \) increases. As \( q \) increases, the value lost due to the type I error increases and a more precise signal is required to offset it. Therefore, the incremental benefit of buyout options decreases as \( q \) increases. In Corollary 3(2), the buyout option becomes more attractive as the firm’s value under replacement \( (V) \) increases because the benefit from screening increases as well. Therefore, the required precision of a signal can decline as \( V \) becomes larger. Finally, in Corollary 3(3) and Corollary 3(4), as \( v_H \) and \( v_L \) increase, the loss of firm value due to worker replacement, based on an incorrect signal, increases. The required precision of a signal then increases and the buyout option becomes less attractive as a result.

Given the result that a buyout contract can improve the firm’s screening performance, one may wonder whether a buyout contract is optimal. To examine whether a buyout contract can be optimal, consider a direct revelation mechanism in which a worker is asked to send a report about a signal \( (\sigma) \) which he receives during job training and compare the performance with the result under the buyout contract.

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The contract \( \{I(\sigma_H), I(\sigma_L), t_S(\sigma_L), t_H(\sigma_H), t_L(\sigma_H)\} \) specifies the exit decision and the compensation. The exit decision \( I \in \{0, 1\} \) is an indicator variable: \( I(\sigma_H) = 0 \) if a worker stays; otherwise, \( I(\sigma_L) = 1 \). The worker’s compensation is \( t_S(\sigma_L) \) if a worker reports \( \sigma_L \), and \( t_H(\sigma_H) \) and \( t_L(\sigma_H) \) if a worker reports \( \sigma_H \) and a high firm value \((v_H)\) is realized and a low firm value \((v_L)\) is realized, respectively. Under the direct revelation mechanism, the firm’s contracting problem can be expressed as follows:

\[
\begin{align*}
\text{Max} & \quad \mu(\lambda(p(v_H - t_H) + (1 - p)(v_L - t_L)) + (1 - \lambda)(V - t_S)) + (1 - \mu)(\lambda(V - t_S) + (1 - \lambda)(v_L - t_L)) \\
\text{s.t.} & \quad \mu(\lambda(pt_H + (1 - p)t_L - c) + (1 - \lambda)t_S) + (1 - \mu)(\lambda t_S + (1 - \lambda)(t_L - c)) \geq 0 \quad (\text{EIR})
\end{align*}
\]

\[
\begin{align*}
t_S(\sigma_L | \sigma_L) & \geq E[u(\sigma_H, e_L) | \sigma_L] \\
E[u(\sigma_H, e_H) | \sigma_H] & \geq t_S(\sigma_L | \sigma_H) \quad (\text{TT-L}) \\
E[u(\sigma_H, e_H) | \sigma_H] & \geq E[u(\sigma_H, e_L) | \sigma_H] \quad (\text{TT-H}) \\
t_S, t_H, t_L \geq 0, \quad (\text{LL})
\end{align*}
\]

where \( \hat{\sigma} \) denotes a reported message and

\[
\begin{align*}
E[u(\hat{\sigma}_H, e_L) | \sigma_L] & = \Pr(\theta_L | \sigma_L)t_L + (1 - \Pr(\theta_L | \sigma_L))(qt_H + (1 - q)t_L) \\
E[u(\hat{\sigma}_H, e_H) | \sigma_H] & = \Pr(\theta_H | \sigma_H)(pt_H + (1 - p)t_L) + (1 - \Pr(\theta_H | \sigma_H))t_L - c \\
E[u(\hat{\sigma}_H, e_L) | \sigma_H] & = \Pr(\theta_H | \sigma_H)(qt_H + (1 - q)t_L) + (1 - \Pr(\theta_H | \sigma_H))t_L.
\end{align*}
\]

(\text{TT-L}) is a truth-telling constraint which ensures that a worker who receives a low-fit signal reports a low-fit signal. Additionally, (\text{TT-H}) induces a worker who receives a high-fit signal to report a message for a high-fit signal. According to (\text{LL}), it is optimal to set \( t_L \) to zero again. In (\text{TT-L}), it is clear that \( t_S(\sigma_L | \sigma_L) = E[u(\hat{\sigma}_H, c_L) | \sigma_L] \) should hold to induce a worker to report a low-fit signal. In addition, to motivate a worker who receives a high-fit signal, (IC-H) should also be binding, too. Given that \( t_L^* = 0 \) and (TT-L) and (IC-H) constraints are
binding, the optimal solutions are as follows:

\[ t^*_L = 0; \quad t^*_H = \frac{c}{\Pr(\theta_H|\sigma_H)(p-q)}; \quad t^*_S = \frac{(1 - \Pr(\theta_L|\sigma_L))cq}{\Pr(\theta_H|\sigma_H)(p-q)}. \]

Accordingly, the firm’s profit is

\[
\Pi_D = \mu p \lambda (v_H - v_L) + (\mu \lambda + (1 - \mu)(1 - \lambda))v_L + V(1 - \mu \lambda - (1 - \mu)(1 - \lambda))
\]

\[ - \frac{c(\lambda p + (1 - \lambda)q)(\mu \lambda + (1 - \mu)(1 - \lambda))}{\lambda(p-q)}, \]

where \( D \) denotes a direct revelation mechanism. If we compare the outcomes with the result under the buyout option contract (Proposition 3), it is true that this equilibrium outcome can be replicated by the buyout option contract. This result confirms that the buyout option contract is optimal because it replicates the result in the direct revelation. Hence it cannot be outperformed by any other mechanisms. This result also implies that the result of communication in a centralized mechanism can be replicated by decentralization of the retention decision to a worker using the buyout option. Proposition 5 summarizes this result.

**Proposition 5** The use of buyout options with job training is optimal in that it cannot be outperformed by any other mechanism.

### 5 Example

This section provides a simple example to compare the preceding cases: benchmark (the first-best), no job training, job training, and job training with buyout options.

Figure 2 compares a firm’s expected profit in each case, with \( v_H = 10, \ v_L = 3, \ V = 5, \ c = 0.2, \ p = 0.7, \ q = 0.6, \) and \( \lambda = 0.8. \) This example delineates how a firm’s value changes as the likelihood that a worker has a high fit (\( \mu \)) varies. As shown in Figure 2, the firm’s profit is always highest under the first-best case (\( \Pi_F \)) when a worker’s fit is observable. If we compare the case with no job training (\( \Pi_N \)) with the case with job training (\( \Pi_J \)), the firm’s value, as suggested previously, increases with job training when the likelihood of a high fit (\( \mu \))
is not extremely high. As this likelihood decreases, screening becomes more important and job training ($\Pi_J$) thus generates a higher profit because it reduces the compensation cost.

Furthermore, the buyout option excludes workers with low-fit signals and increases screening efficiency; the use of buyout options improves a firm’s profit ($\Pi_B$) when $\mu < 0.79$, compared with job training only ($\Pi_J$). However, this strong result for the buyout option assumes relatively high precision of the job training signal. As Proposition 4 shows, the buyout option generates a higher firm’s profit with more precise signals; otherwise, job training alone provides a firm with greater profit. However, when the likelihood of a high fit is great enough, the firm does not have to worry about screening, and no screening mechanism ($\Pi_N$) can be better than job training.

Figure 3 focuses on a comparison between job training alone and job training with buyout options. This example maintains the same parameter values and lets $\mu$ take a value of 0.55. The comparison centers on how the two cases generate different expected profits as the precision of the signal ($\lambda$) changes. As the graph shows, if the signal precision is not too low, the buyout options always generate a higher expected profit by excluding low-fit workers. However, if the
signal precision is too low, the buyout options can be too aggressive in screening and eliminate even workers with a high fit, such that type I error increases. In this case, as Figure 3 shows, job training without buyout options performs better than job training with buyout options.

6 Extensions

This section considers variations such as the endogenous firm value and the effect of skill learning during job training to verify the robustness of the main result in the previous section. The results show that the main results of job training and buyout options are robust to these variations.

6.1 Skill Learning

Another important value of job training is that a new worker can learn a required new skill in a firm and thereby raise the firm’s productivity. However, to focus on the effect of the fit, the main setup does not consider the effect of skill learning. Therefore, this section considers the effect of skill learning on the firm’s productivity and screening process and shows whether the main results still hold even after considering the effect of skill learning.

The effect of skill learning on productivity is defined as $\tau$. That is, the likelihood of a high firm value ($v_H$) increases by $\tau$ after the job training when a worker exerts high effort. Considering the effect of $\tau$, the chance of a high (low) firm value is as follows:

\[
\Pr(v_H|\theta, e, \tau) = \begin{cases} (p + \tau)e + q(1 - e) & \text{if } \theta = \theta_H \\ \tau e + 0 \cdot (1 - e) = \tau e & \text{if } \theta = \theta_L \end{cases};
\]
\[
\Pr(v_L|\theta, e, \tau) = 1 - \Pr(v_H|\theta, e, \tau),
\]

where $p + \tau \in \left[\frac{1}{2}, 1\right]$.

In this probability structure, a significant change from the previous main setup is that even a low-fit worker can generate $v_H$ with the probability, $\tau$ if he exerts high effort. However, if the worker does not exert high effort, the skill does not affect the firm value regardless of his fit.
When the effect of skill learning is considered, it is clear that the advantage of job training and buyout options over no job training will increase because the skill learning factor introduces additional productivity, as denoted by $\tau$. Furthermore, one may expect that this consideration of skill learning lowers the attractiveness of buyout options; even without replacement, the remaining low-fit worker can generate a high firm value ($v_H$) if he exerts high effort. However, unlike the expected outcome, the benefit of a buyout option does not decrease and its relative advantage over the case with job training only case is identical to that in the main setup. This result arises because the additional productivity due to $\tau$ allows a firm to lower a worker’s compensation ($t^*_H = \frac{c}{Pr(\theta_H|\sigma_H)(p-q)+\tau}; t^*_L = 0$). In such a case, the predetermined buyout price also decreases because the information rent of the staying low-fit-signal worker is reduced by the lower compensation. The advantage from the lower buyout price offsets the benefit of the case with job training only, which arises from the additional productivity due to skill learning. This result is summarized in Proposition 6.

**Proposition 6**

(1) When the skill is considered, the optimal buyout price is

$$K^* = \frac{cq(1-Pr(\theta_L|\sigma_L))}{Pr(\theta_H|\sigma_H)(p-q)+\tau}.$$  

The optimal buyout price decreases as skill learning ($\tau$) increases.

(2) The relative benefit of a buyout option over the case with job training only does not change, even after skill learning is considered.

As shown in Proposition 6, the buyout price ($K^*$) decreases as $\tau$ increases, thus reducing a firm’s screening cost. As the skill improves a firm’s productivity, the firm can pay less compensation to motivate the worker to exert high effort because the chance for a high firm value is higher. The lower compensation leads to lower information rent for a low-fit-signal worker, thus reducing the incentive for separation: remaining in the firm becomes less attractive for a low-fit-signal worker under a situation with additional productivity due to skill learning. Thus, the result confirms that the benefit of a buyout option is robust to the consideration of skill learning during job training. This result also suggests that job training is different from a simple information system and that it can function as a mechanism that raises a
firm’s productivity by improving workers’ skill level while providing workers with the private information.

6.2 Endogenous \( V \)

In the main setup, if a worker leaves, the firm finds a new worker and thus realizes an exogenous firm value \( V \in (v_L,v_H) \). For simplicity, the firm value is set to \( V \) exogenously in the main setup. One may wonder whether the benefit of buyout options continues to exist even after the exogenous firm value is endogenized. This section considers the endogenous firm value generated by a replaced worker and examines whether the main result of job training and buyout options still holds.

For the replacement, the firm finds a new worker from the same pool of workers, and the likelihood that a replaced worker has a high fit is \( \mu \), as it was before. In this case, the expected firm value from the replacement is

\[
V_E = \mu (p(v_H - t_H) + (1 - p)(v_L - t_L)) + (1 - \mu)(v_L - t_L),
\]

where \( E \) denotes the endogenous firm value. By substituting \( V_E \) for \( V \) in the firm’s objective function under the buyout option, the optimal solutions are derived. This result is summarized in Corollary 4.

**Corollary 4** Under the endogenous \( V \), when \( \mu \leq \mu_E = \frac{p+q}{2p}, \) the use of a buyout option with job training improves a firm’s profit compared to the case with job training only if \( \lambda \geq \lambda_E \).

As shown in the result, the benefit of buyout options still holds even when we consider the endogenous firm value generated by a replaced worker. When the firm replaces a worker, it has an expectation for the new worker’s productivity based on the pool of workers. As long as the precision of job training is not too low, the expected productivity is higher than a low-fit-signal worker’s productivity and the buyout option is a profitable strategy for a firm. The result based on the endogenous firm value emphasizes again that the common concern among practitioners that losing skilled workers by labor buyouts may lead to a firm’s lower productivity in the long run may be mitigated, as ousting workers through buyouts can be
accompanied by the "acquisition of more productive new workers on average" compared to the
sure “low fit” workers who left because the new workers may have a high fit. Thus Corollary
4 confirms that the main benefit of buyout options is robust to an endogenous $V$.

7 Conclusion

This study examines how job training combined with buyout options may enhance a firm’s
screening efficiency when its fit with a potential worker is not observable at the contracting
stage. Without job training, neither the worker nor the firm can learn about the fit at the
contracting stage, and the firm must design an inefficient contract that motivates both high-
and low-fit workers. Job training offers an opportunity for a worker to learn about his fit before
exerting effort on the job. The knowledge of fit is a worker’s private information, but the firm
can still improve its contracting efficiency with this private information because it can design
a selective compensation scheme that motivates only a worker with a high-fit signal and thus
minimize expected compensation costs.

Even with job training, a worker with a low-fit signal does not want to leave because he
can earn information rent by staying and exerting no effort, but the buyout option serves as an
incentive device that induces workers with a low-fit signal to leave voluntarily. The worker can
take the buyout option only after job training and the option expires immediately thereafter.
Hence, a worker observing a low-fit signal from job training should take the buyout option
and leave because the buyout price provides a greater payoff than expected compensation
from staying. Despite these incentives granted to the leaving worker, a firm’s expected profit
improves because the firm can replace the low-fit worker with a more productive new worker
on average. Because the signal from job training is imperfect, however, the buyout option may
also be too aggressive in removing workers who receive low-fit signals, which constitutes a type
I error. The final result shows that the firm can increase its profit when the likelihood of a
high-fit worker is not extremely high because the benefit from excluding workers with low-fit
signals dominates the loss from type I error. In addition, it is shown that the benefit of job training and buyout options is robust to the effect of skill learning and an endogenous firm value.

This study offers two novel results. The conventional wisdom is that a worker’s private information causes a disadvantage to a firm by creating information asymmetry. However, this paper shows a firm needs to help a worker learn his private information through job training, thereby allowing a firm to design a more efficient compensation scheme.

In addition, it has long been recognized that a guaranteed payment in the form of a fixed payment cannot work as an incentive device, but this study suggests that a fixed buyout price can serve as a contracting tool when the hidden knowledge problem exists at the interim stage.

As a caveat, the research setting does not consider the cost of designing the job training program. In practice, designing an effective job training program requires significant investment. Therefore, when the job training is more costly, the benefit of job training will decline. Further extension of this research as regards this topic could consider this feature.
8 Appendix

Proof of Lemma 1

In equilibrium, the (IC) constraint should be binding as follows:

\[(p - q)t_L = (p - q)t_H - c\]
\[\iff t_H = t_L + \frac{c}{p - q}.\]

Due to the limited liability assumption, it is always optimal to set \(t^*_L = 0\). Therefore, \(t^*_L = 0\) and \(t^*_H = \frac{c}{p - q}\). All constraints are satisfied by the equilibrium solutions and the firm’s profit is as follows:

\[\Pi_F = \mu \left( pv_H + (1 - p)v_L - \frac{c}{p - q} \right) + (1 - \mu)V \quad Q.E.D.\]

Proof of Proposition 1

(1) By the binding (IC) constraint and the (LL) constraint,

\[t^*_L = 0; \ t_H = \frac{c}{\mu(p - q)}.\]

which also satisfy the (EIR) constraint.

The solutions yield a following firm’s profit:

\[\Pi_N = \mu(pv_H + (1 - p)v_L) + (1 - \mu)v_L - \frac{cp}{p - q}.\]

(2) If we compare \(\Pi_N\) with \(\Pi_F\),

\[\Pi_N - \Pi_F = -\frac{(1 - \mu)((p - q)(V - v_L) + cp)}{p - q} < 0. \quad Q.E.D.\]

Proof of Proposition 2

(1) First, it is shown that a low-fit signal worker with \(\sigma_L\) always exerts low effort in equilibrium.

The low-fit signal worker’s expected payoff based on the chosen effort level is as follows:

\[E[u|\sigma_L, c_H] = \Pr(\theta_L|\sigma_L)t_L + (1 - \Pr(\theta_L|\sigma_L))(pt_H + (1 - p)t_L) - c\]
\[E[u|\sigma_L, c_L] = \Pr(\theta_L|\sigma_L)q_L + (1 - \Pr(\theta_L|\sigma_L))(qt_H + (1 - q)t_L).\]
For the low-fit signal worker to exert high effort, a following condition should be satisfied:

\[
E[u|\sigma_L, c_H] - E[u|\sigma_L, c_L] = (1 - \Pr(\theta_L|\sigma_L))(p - q)(t_H - t_L) - c > 0
\]

\[
t_H - t_L > \frac{c}{(1 - \Pr(\theta_L|\sigma_L))(p - q)} (*).
\]

By the binding (IIC-H) and the limited liability assumption,

\[
t^*_L = 0; \quad t^*_H = \frac{c}{\Pr(\theta_H|\sigma_H)(p - q)} = \frac{c(\mu\lambda + (1 - \mu)(1 - \lambda))}{\mu\lambda(p - q)}.
\]

Then,

\[
t^*_H - t^*_L = \frac{c}{\Pr(\theta_H|\sigma_H)(p - q)} < \frac{c}{(1 - \Pr(\theta_L|\sigma_L))(p - q)} (*),
\]

which implies that the condition (*) for the low fit signal worker to exert high effort is not satisfied in equilibrium and he always exerts low effort in equilibrium.

The optimal solutions, \(t^*_L\) and \(t^*_H\), satisfy the (EIR) and (IIR-H) constraints and

\[
E[u|\sigma_L] = \frac{cq(1 - \lambda)(\mu\lambda + (1 - \mu)(1 - \lambda))}{(p - q)\lambda(1 - \mu\lambda - (1 - \mu)(1 - \lambda))} > 0,
\]

which implies that a low-fit signal worker does not leave a firm because he can earn more than the reservation utility (zero) by staying in a firm.

The equilibrium outcomes yield a following firm’s profit:

\[
\Pi_J = \mu(\lambda(p(v_H - t_H) + (1 - p)(v_L - t_L)) + (1 - \lambda)(q(v_H - t_H) + (1 - q)(v_L - t_L))) + (1 - \mu)t_L
\]

\[
= \mu(\lambda p + (1 - \lambda)q)(v_H - v_L) + v_L - \frac{c(\lambda p + (1 - \lambda)q)(\mu\lambda + (1 - \mu)(1 - \lambda))}{\lambda(p - q)}.
\]

(2) If we compare \(\Pi_J\) with \(\Pi_N\), \(\Pi_J > \Pi_N\)

\[
\text{if } \mu \leq \mu^* = \frac{c(p\lambda^2 - q(1 - \lambda)^2)}{(p - q)^2(v_H - v_L)(1 - \lambda)\lambda + c(2\lambda - 1)(p\lambda + q(1 - \lambda))}. \quad \text{Q.E.D.}
\]

Proof of Corollary 1

\[
\frac{\partial E[u|\sigma_L]}{\partial \lambda} = -\frac{cq(1 - \mu)(\mu + 2\lambda(1 - \lambda)(1 - 2\mu))}{(p - q)\lambda^2(\mu + \lambda - 2\mu\lambda)^2} < 0. \quad \text{Q.E.D.}
\]
Proof of Proposition 3

By the (IC) constraint and the (LL) constraint, the optimal solutions are:

\[ t^*_L = 0; \quad t^*_H = \frac{c(\mu\lambda + (1 - \mu)(1 - \lambda))}{\mu\lambda(p - q)}. \]

As shown in the objective function, as \( K \) becomes smaller, a firm’s profit becomes greater. Then, the optimal value for the buyout price, \( K^* \) is \( E[u|\sigma_L, c_L] = \Pr(\theta_L|\sigma_L)t_L + (1 - \Pr(\theta_L|\sigma_L))(qt_H + (1 - q)t_L) \) because it maximizes a firm’s profit while satisfying both the (EXIT-L) and the (NoEXIT-H) constraints. Then, the optimal buyout price is

\[ K^* = \frac{cq(1 - \Pr(\theta_L|\sigma_L))}{\Pr(\theta_H|\sigma_H)(p - q)} = \frac{cq(1 - \lambda)(\mu\lambda + (1 - \mu)(1 - \lambda))}{(p - q)\lambda(1 - \mu\lambda - (1 - \mu)(1 - \lambda))}. \]

If we put \( t^*_L \) and \( t^*_H \) in the following constraints,

\[ \frac{cq(1 - \mu + \lambda(2\mu - 1))}{p - q} > 0 \quad \text{(EIR)} \]
\[ \frac{cq}{p - q} > 0 \quad \text{(IIR-H)} \]
\[ \frac{cq}{p - q} > K^* = \frac{cq(1 - \Pr(\theta_L|\sigma_L))}{\Pr(\theta_H|\sigma_H)(p - q)} = \frac{cq(1 - \lambda)(\mu\lambda + (1 - \mu)(1 - \lambda))}{(p - q)\lambda(1 - \mu\lambda - (1 - \mu)(1 - \lambda))} \quad \text{(NoEXIT-H)}, \]

which imply that all the constraints are satisfied by the optimal solutions. The optimal solutions yield a following firm’s profit:

\[ \Pi_B = \mu p\lambda(v_H - v_L) + (\mu\lambda + (1 - \mu)(1 - \lambda))v_L + V(1 - \mu\lambda - (1 - \mu)(1 - \lambda)) \]
\[ -\frac{c(\lambda p + (1 - \lambda)q)(\mu\lambda + (1 - \mu)(1 - \lambda))}{\lambda(p - q)}. \quad Q.E.D. \]

Proof of Corollary 2

As shown above, \( K^* = \frac{cq(1 - \Pr(\theta_L|\sigma_L))}{\Pr(\theta_H|\sigma_H)(p - q)} = \frac{cq(1 - \lambda)(\mu\lambda + (1 - \mu)(1 - \lambda))}{(p - q)\lambda(1 - \mu\lambda - (1 - \mu)(1 - \lambda))} \). Also, \( \Pr(\theta_L|\sigma_L) \) and \( \Pr(\theta_H|\sigma_H) \) are a function of only \( \mu \) and \( \lambda \).

(1) \( \lambda \)

\[ \frac{\partial K^*}{\partial \lambda} = -\frac{cq(1 - \mu)(\mu + 2(1 - \lambda)\lambda(1 - 2\mu))}{\lambda^2(p - q)(\mu + \lambda - 2\mu\lambda)^2} < 0. \]

(2) \( p \)

\[ \frac{\partial K^*}{\partial p} = \frac{-cq(1 - \Pr(\theta_L|\sigma_L))}{\Pr(\theta_H|\sigma_H)(p - q)^2} < 0. \]
Proof of Proposition 4

(1) The comparison between a firm’s profit under job training with buyout options ($\Pi_B$) and a firm’s profit without job training ($\Pi_N$) is as follows:

$$\Pi_B - \Pi_N = (V - v_L)\lambda - (p(1 - \lambda)(v_H - v_L) + (2\lambda - 1)(V - v_L))\mu + \frac{c(\lambda(p + \mu - 2\mu\lambda) - q(1 - \lambda)(1 - \mu - \lambda + 2\mu\lambda)}{\lambda(p - q)} \geq 0$$

if $\mu \leq \mu^B = \frac{\lambda^2(p - q)(V - v_L + c) + cq(2\lambda - 1)}{c(2\lambda - 1)(\lambda p + (1 - \lambda)q) + \lambda(p - q)((1 - \lambda)p(v_H - v_L) + (2\lambda - 1)(V - v_L))}.$

(2) The comparison between a firm’s profit under job training with buyout options ($\Pi_B$) and a firm’s profit under job training only ($\Pi_J$) is as follows:

$$\Pi_B - \Pi_J = V((1 - \mu)\lambda + \mu(1 - \lambda)) - (\mu(1 - \lambda)q(v_H - v_L) + ((1 - \mu)\lambda + \mu(1 - \lambda))v_L)$$

if $\lambda > \lambda^* = \frac{\mu(qv_H + (1 - q)v_L - V)}{\mu(qv_H + (1 - q)v_L - V) + (V - v_L)(1 - \mu)}.$

(3) If we compare $\mu^B$ with $\mu^J$

$$\mu^B - \mu^J \geq 0 \text{ if } \lambda > \lambda^* = \frac{\mu(qv_H + (1 - q)v_L - V)}{\mu(qv_H + (1 - q)v_L - V) + (V - v_L)(1 - \mu)}.$$

Proof of Corollary 3

(1) $q$

$$\frac{\partial \lambda^*}{\partial q} = \frac{(1 - \mu)\mu(V - v_L)(v_H - v_L)}{(\mu(qv_H + (1 - q)v_L - V) + (V - v_L)(1 - \mu))^2} > 0.$$

(2) $\lambda$

$$\frac{\partial \lambda^*}{\partial V} = \frac{(1 - \mu)\mu q(v_H - v_L)}{(\mu(qv_H + (1 - q)v_L - V) + (V - v_L)(1 - \mu))^2} < 0.$$

(3) $v_H$

$$\frac{\partial \lambda^*}{\partial v_H} = \frac{(1 - \mu)\mu q(V - v_L)}{(\mu(qv_H + (1 - q)v_L - V) + (V - v_L)(1 - \mu))^2} > 0.$$
Proof of Proposition 6

(1) First, under the job training only case, by the (IC) constraint and the (LL) constraint, the optimal solutions are:

\[ t^*_L = 0; \quad t^*_H = \frac{c}{\Pr(\theta_H|\sigma_H)(p - q) + \tau}. \]

By the optimal compensations,

\[ \Pi_J = v_L + \frac{((p+q-q)\mu+\tau-(\mu+\lambda(1-2\mu))\tau)(c(\lambda+\mu-1-2\mu)+(v_H-v_L)(\lambda\mu(p-q)+(1-\lambda-\mu+2\mu)\tau))}{\lambda\mu(p-q)+(1-\lambda-\mu+2\mu)\tau}. \]

Under the buyout option, the optimal compensations, \( t^*_L \) and \( t^*_H \) are the same as before. The optimal buyout price, \( K^* \) is \( E[u|\sigma_L, c_L] = \Pr(\theta_L|\sigma_L)t_L+(1-\Pr(\theta_L|\sigma_L))(qt_H+(1-q)t_L) \) because it maximizes a firm’s profit while satisfying both the (EXIT-L) and the (NoEXIT-H) constraints. Then, the optimal buyout price is

\[ K^* = \frac{cq(1-\Pr(\theta_L|\sigma_L))}{\Pr(\theta_H|\sigma_H)(p - q) + \tau}. \]

It is straightforward to see that \( K^* \) decreases as \( \tau \) increases and a firm’s profit is:

\[
\Pi_B = v_L(1-\lambda-\mu) + \lambda\mu(p(v_H-v_L) + 2v_L) + V(\lambda + \mu - 2\lambda\mu) + (v_H - v_L)(1 - \mu + \lambda(2\mu - 1))\tau - c(1 - \mu + \lambda(2\mu - 1))(1 + \frac{q\mu}{\lambda\mu(p-q)+(1-\lambda-\mu+2\mu)\tau}).
\]

(2) If we compare \( \Pi_B \) with \( \Pi_J \),

\[
\Pi_B - \Pi_J = V((1-\mu)\lambda + \mu(1-\lambda)) - (\mu(1-\lambda)q(v_H-v_L)+((1-\mu)\lambda+\mu(1-\lambda))v_L)
\]

if \( \lambda > \lambda_s = \frac{\mu(qv_H+(1-q)v_L-V)}{\mu(qv_H+(1-q)V-L)+(V-v_L)(1-\mu)}, \)

where \( s \) denotes skill learning. If we compare \( \lambda_s \) with \( \lambda^* \) in Proposition 4, they are identical and the result implies that the benefit of buyout options does not change even after skill learning is considered. Q.E.D.
Proof of Corollary 4

\[ \Pi_B - \Pi_J = \frac{-(p-q)(v_H - v_L)\lambda \mu (q(1-\lambda) + 2p\lambda \mu - p(\lambda + \mu))}{(p-q)\lambda} \]

\[ - \frac{c\mu \lambda (1-2\mu)}{(p-q)\lambda} (1-\mu + \lambda(-1 + 2\mu)). \]

\[ \frac{\partial (\Pi_B - \Pi_J)}{\partial \lambda} = \frac{(v_H - v_L)\lambda^2 \mu (p^2 - q^2 - 2p(p-q)\mu) + c\mu \lambda^2 (1 - 2\mu)^2 + \mu - \mu^2)}{(p-q)\lambda^2} > 0 \text{ if } \mu \geq \frac{p+q}{2p}. \]

When \( \lambda = 1 \),

\[ \Pi_B - \Pi_J = \frac{p((p-q)(v_H - v_L) - c)(1 - \mu)\mu}{p-q} > 0. \]

Then there exists \( \lambda_E \) which makes \( \Pi_B \) equal to \( \Pi_J \). \( Q.E.D. \)
References


